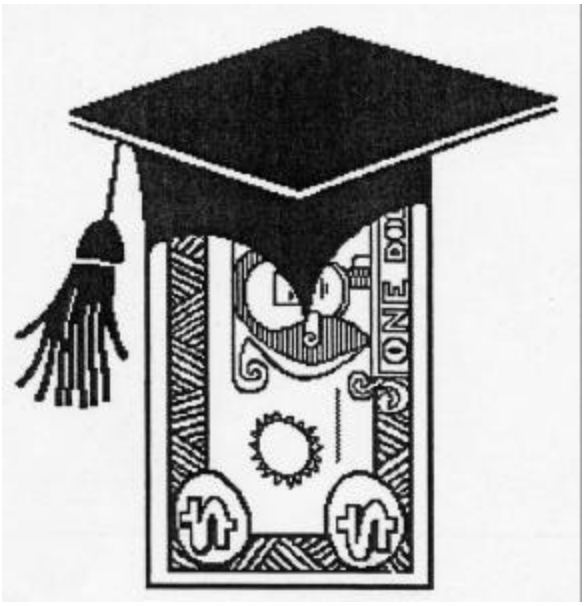
Algebra



This algebra section is a collection twelve sets of algebra problems that are similar to those found on SAT/PSAT tests.

1. Find the value of an expression
2. Solving equations
3. Equations that must be solved for more than a single variable
4. Formulating your own equation and solving it
5. Problems involving percent
6. Inequalities
7. Factors and factoring
8. Ratio and proportion
9. Simplifying expressions
10. Average
11. Problems involving systems of equations
12. Slope

- When an Algebra 1 teacher gives a test that covers any of these concepts, we recommend that the teacher distribute and assign the corresponding set of problems from above which the student should complete by the day following the test.

#1 Find the value of an expression

3(7) - 4 is an example of a numerical expression.
The value of this expression is 17.
A numerical expression contains no variables.

3x + 5 is an example of an algebraic expression.
The value of this expression depends on what value is assigned to **x**. For example, if **x** is given a value of 9, the value of the whole expression is 32. 3(9) + 5 = 27 + 5 = 32

In order to find the value of an expression, you must substitute in the values of the variables and simplify.

It is easy to make mistakes when you are simplifying.
The best way to avoid mistakes is to **always use parentheses.**

1. If x = 2 and y = -3, what is the value of 3x - 4y?
(A) -6 (B) -2 (C) 12 (D) 14 (E) 18

2. If x = -2, and y = -3, which of the following is the greatest?
(A) $\frac{1}{x} + \frac{1}{y}$ (B) 1+ x + y (C) 1 - x - y (D) $\frac{x^2}{2} + \frac{y^2}{3}$ (E) $x^3 + y^3$

A problem becomes more difficult if a variable is equal to a fraction as shown in #3.

3. If x = $\frac{-1}{2}$, which of the following is the greatest?
(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{1}{x^3}$ (D) 1 - x (E) x^2

Fractions frighten many fine students, yet fractions will be your friends if you remember these three magic words: ***Write it horizontally.***

This means that if you ever have a fraction divided by a fraction, a fraction divided by a whole number, or a whole number divided by a fraction, rewrite the problem horizontally.

In #3, after substituting in choice (A), it will say $\frac{1}{\frac{-1}{2}}$ } Rewrite this as $1 \div \frac{-1}{2}$, which equals $1 (\frac{2}{-1}) = -2$.

4. If $x = 3$, then $(2x)^2 - 2x^2 =$
(A) $x - 6$ (B) 0 (C) 6 (D) 18 (E) 24

5. If $3x = 3x^3$, what value(s) of x satisfy the equation?
(A) 0 only (B) 1 only (C) 0 or 1 only (D) -1 or 1 only (E) -1, 0 or 1

6. If Set A consists of (1, 3, 5, 7, 9, 11) and Set B consists of (2, 4, 6, 8, 10, 12) and Set C consists of (5, 10, 15, 20), what is the greatest possible value of

Note: An element is a member of a set.

$$\frac{(\text{element of Set A})(\text{element of Set C})}{(\text{element of Set B})} \; ?$$

(A) $2\frac{1}{2}$ (B) 3 (C) $18\frac{1}{3}$ (D) 110 (E) 220

7. Which of the following would NOT result in a value of 12 for the expression $(x)(y) - z$?
(A) $x = 1, y = 10, z = -2$
(B) $x = 7, y = 2, z = 2$
(C) $x = 4, y = 4, z = 4$
(D) $x = 4, y = 3, z = 0$
(E) $x = 6, y = 2, z = 0$

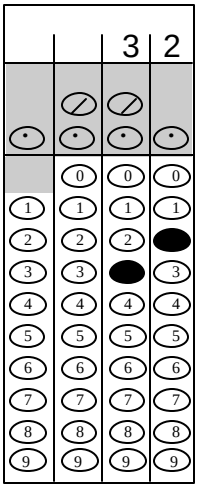
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

8. If $x = 4$, then $(2x)^2 - 2x^2 =$

9. If $a - b = -4$, then $-2(a - b)(a - b) =$

10. If $a - b = 3$, then $(a - b)(b - a) =$

11. If $x = 6$, then 30% of $15x =$



Answers: 1. E 2. C 3. B 4. D 5. E 6. D 7. D
8. 32 9. -32 10. -9 11. 27

#2

Solving equations

Remember the following information:

1) The SAT and PSAT include questions that involve solving an equation.
2) For multiple-choice questions, the answer is often right there in front of you as one of the five answer choices.
3) In a multiple-choice question, if you are have difficulty solving an equation, insert each of the answer choices into the given equation until you find the choice that works.
4) However, if you just solve the equation as shown below, make sure that you check your answer by substituting it into the original equation.

Example: What is the value of x if $x - 4 = 10 - x$?

(A) -7 (B) -6 (C) 6 (D) 7 (E) 14

Solving the problem is shown to the right:

$$x - 4 = 10 - x$$
$$+x \quad = \quad +x$$
$$2x - 4 = 10$$
$$+4 = +4$$
$$2x = 14$$

$$\frac{2x}{2} = \frac{14}{2}$$

$x = 7$

which is (D)

If you sense that solving an equation is going to be difficult for you, substitute each of the answer choices into the given equation until you find the choice that works.

This allows you to solve the question with arithmetic instead of algebra.

Now, please check.

$x - 4 = 10 - x$

$(7) - 4 = 10 - (7)$

$3 = 3$

A) $-7 - 4 = 10 - (-7)$
$-11 = 17$, false

B) $-6 - 4 = 10 - (-6)$
$-10 = 16$, false

C) $6 - 4 = 10 - 6$
$2 = 4$, false

D) $7 - 4 = 10 - 7$
$3 = 3$, **true**

E) $14 - 4 = 10 - 14$
$10 = -4$, false

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1. If $x - 9 = 3 - x$, then $x = ?$
(A) -6 (B) -3 (C) 3 (D) 6 (E) 12

2. If $(33)(3)(y) = 11$, then $y = ?$

(A) $\frac{1}{11}$ (B) $\frac{1}{10}$ (C) $\frac{1}{9}$ (D) 9 (E) 10

3. If $\frac{1}{x} + \frac{1}{x} = 8$, then $x = ?$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 4 (E) 8

4. If $\frac{(18)(6)}{n} = 9$, then $n = ?$

(A) $\frac{1}{12}$ (B) 6 (C) 9 (D) 12 (E) 18

5. If $x + .5 + .25 = 1$, then $x = ?$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{7}{10}$ (D) $\frac{3}{4}$ (E) $\frac{7}{4}$

6. If $(1 + 3)(y + 4) = 24$, then $y = ?$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. If $\frac{(10 + 30) + (y + 50)}{3} = 30$, then $y = ?$

(A) 0 (B) 10 (C) 20 (D) 30 (E) 40

8. If $\frac{(10 + 30) - (y - 50)}{4} = 20$, then $y = ?$

(A) -90 (B) -10 (C) 0 (D) 10 (E) 90

9. If $y \div 7 = 402 \div 6$, then $y = ?$

(A) 6 (B) 9 (C) $9\frac{4}{7}$ (D) 67 (E) 469

10. If $\frac{y}{3} = 6$ and $\frac{3}{x} = 1$, then $\frac{2 + x}{2 + y} = ?$

(A) $\frac{1}{4}$ (B) 1 (C) $\frac{7}{6}$ (D) $\frac{5}{4}$ (E) 4

11. If $\frac{4}{5} = \frac{x}{4}$, then x = ?

(A) $\frac{5}{16}$ (B) $\frac{5}{4}$ (C) $\frac{16}{5}$ (D) 5 (E) 16

12. If $2\frac{5}{6} = 1 + \frac{y}{24}$, then y = ?

(A) 20 (B) 29 (C) 30 (D) 44 (E) 68

A detailed explanation is given for problems 13 and 14 below.

13. If $\frac{2}{3}x = \frac{3}{4}x$, what is the value of x ?

(A) 0 (B) $\frac{8}{9}$ (C) $\frac{9}{8}$ (D) $\frac{4}{3}$ (E) It cannot be determined from the information given.

14. If $\frac{2}{3}x = \frac{4}{6}x$, what is the value of x?

(A) 0 only (B) $\frac{3}{8}$ only (C) 1 only (D) $\frac{3}{2}$ only

(E) All real numbers

Problems 13 and 14 look similar but are different.

#13 can be solved as follows:

$$\frac{2}{3}x = \frac{3}{4}x$$

First, to eliminate the fractions, multiply by the lowest common denominator, 12.

$$12\left(\frac{2}{3}x\right) = 12\left(\frac{3}{4}x\right)$$

This will give

$$8x = 9x$$

Now subtract 8x from both sides of the equation, and you will get

$$0 = x$$

#14 can be solved as follows:

$$\frac{2}{3}x = \frac{4}{6}x$$

First, to eliminate the fractions, multiply by the lowest common denominator, 6.

$$6\left(\frac{2}{3}x\right) = 6\left(\frac{4}{6}x\right)$$

This will give

$$4x = 4x$$

Now subtract 4x from both sides of the equation, and you will get

$$0 = 0$$

This is a different situation from #13. Read below to see why

If, while solving an equation, the variables disappear, there are two possible answers. If you are left with a true statment, as in the case in #14 (0 = 0), the answer is all real numbers. This means that any number will work. So the answer to #14 is (E). If you are left with a false statement, like 3 = 2, that means that no answer will work or ∅.

15. If $\frac{x + 2}{x - 1} = 4$, what is the value of x?

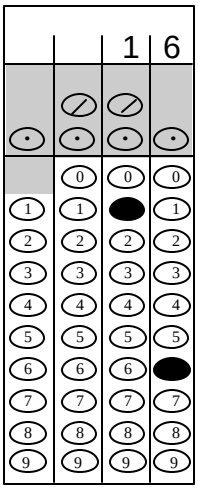
(A) -1 (B) 1 (C) 2 (D) 3 (E) 4

16. If $2(x - 4) + 3(x - 4) + 4(x - 4) = 0$, what is the value of x?

(A) -4 (B) -4/3 (C) 0 (D) 4/3 (E) 4

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

17. If x/3 - 1/3 = 5, what is the value of x?



18. If 3y + 5z = 25, and y = x + 1, and z = x - 2, what is the value of x?

19. If 1/x = 3/5, what is the value of x?

20. 3y + 2z = xy + xz
If y = 2 and z = 3, what is the value of x?

21. If $2^{4x} = 2^{2x + 2}$, what is the value of x?

22. If $2^{3x} = 4^{2x - 1}$, what is the value of x?

23. For what value of x is x + 3x + 5x = 10?

Answers: 1. D 2. C 3. B 4. D 5. A 6. C 7. A 8. D 9. E 10. A 11. C 12. D 13. A 14. E 15. C 16. E 17. 16 18. 4 19. 5/3 20. 12/5 21. 1 22. 2 23. 10/9

#3 Equations that must be solved for more than a single variable

It is common to have another type of problem on PSATs and SATs that involves solving equations.

In this type of problem you can usually get the correct answer without solving for the individual variables. The following example will demonstrate.

If $2x - 4 = 20$, then $\frac{2x - 4}{2} = ?$

The long way to do this problem is to solve for x in the first equation and then substitute the result into the second equation for x.

$2x - 4 = 20$

$+4 = +4$

$2x = 24$

$\frac{2x}{2} = \frac{24}{2}$

$x = 12$

Substitute this into $\frac{2x - 4}{2}$

$\frac{2(12) - 4}{2}$

$\frac{24 - 4}{2} = \frac{20}{2} = 10$

The short way to do the problem is to see that if you divide both sides of the original equation by 2, you will achieve the answer.

$2x - 4 = 20$

$\frac{2x - 4}{2} = \frac{20}{2} = 10$

If you are asked to find more than a single variable, there is usually a shorter and simpler method to reach the answer.

1. If $2x - 4 = 6a$, then $\frac{2x - 4}{2} = ?$

(A) $\frac{1}{2}a$ (B) a (C) $2a$ (D) $3a$ (E) $4a$

2. If $18x - 12y = 5$, then $6x - 4y = ?$

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{3}{5}$ (D) $\frac{5}{3}$ (E) 3

3. If $(2x)\left(\frac{3}{5}\right) = \left(\frac{3}{5}\right)\left(\frac{3}{4}\right)$, then $2x = ?$

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) 3 (D) 5 (E) 9

4. If $\frac{3}{4}x = 1$, then $\frac{1}{4}x = ?$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{4}{3}$ (D) 1 (E) 3

5. If $3r - 1 = 4$, then $6r = ?$

(A) 6 (B) 10 (C) 12 (D) 18 (E) 20

6. If $a + b = 10$, then $3a + 3b = ?$

(A) $\frac{10}{3}$ (B) 10 (C) 30 (D) 60 (E) 1,000

7. If $\frac{5y}{3} = 4$, then $10y = ?$

(A) $\frac{3}{2}$ (B) $\frac{25}{6}$ (C) 24 (D) 40 (E) 120

8. If $3x + 4y = 10$ and $2x + 3y = 6$, then $x + y = ?$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 4

The key to doing this problem quickly is to subract the second equation from the first.

$$\begin{array}{r} 3x + 4y = 10 \\ -2x - 3y = -6 \\ \hline x + y = 4 \end{array}$$

9. If $3x$ is 2 more than $3y$, then $x - y = ?$

(A) -2 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) $-\frac{2}{3}$

First, write an equation based on the given information.

$$3x = 3y + 2$$

Second, subtract $3y$ from both sides.

$$\begin{array}{r} -3y = -3y \\ \hline 3x - 3y = 2 \end{array}$$

Third, to get $x - y$, divide both sides of the equation by 3.

$$\frac{3x - 3y}{3} = \frac{2}{3}$$

The result is $x - y = \frac{2}{3}$

Answers: 1. D 2. D 3. A 4. B 5. B 6. C 7. C 8. E 9. B

#4 Formulating your own equation and solving it

In order to solve the following problems,
you must set up an equation and solve it.

1. The members of a club raised money selling flowers. Each member of the club sold 4 sets of flowers and charged $2 per set. One lady who bought some flowers donated $5 extra to the club. If, including the donation, a total of $77 was collected, how many members were in the club?

(A) 7 (B) 8 (C) 9 (D) 11 (E) 18

2. If $\frac{1}{3}$ of a number is 3 less than $\frac{1}{2}$ of the number, what is the number?

(A) -18 (B) 0 (C) 3 (D) 9 (E) 18

3. At a high school, $\frac{1}{2}$ of the students are Black, $\frac{1}{4}$ of the students are Anglo, $\frac{1}{6}$ of the students are Hispanic, and there are 10 other students. How many students are there in the school?

(A) 10 (B) 20 (C) 40 (D) 120 (E) 240

4. A man bought some stock at $24.75 per share and sold it at $24.55 per share. If he lost a total of $10, how many shares of stock were sold?

(A) 10 (B) 20 (C) 50 (D) 100 (E) 500

5. There was some money in a cookie jar. After Joe took $\frac{1}{4}$ of the money, there was $48 left. How much money was in the cookie jar before Joe took any?

(A) $12 (B) $36 (C) $60 (D) $64 (E) $192

6. After Mary gave Wendy $6, she still had $6 more than Wendy. How much more money than Wendy did Mary originally have?

(A) $6 (B) $12 (C) $18 (D) $24
(E) It cannot be determined from the information given.

This next problem looks similar to #6 but is a little different.

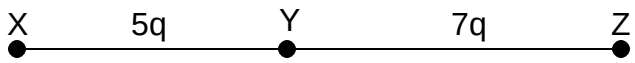
7. After Mary gave Wendy $6, she still had $6 more than Wendy. How much money did Mary originally have?
(A) $6 (B) $12 (C) $18 (D) $24
(E) It cannot be determined from the information given.

8. 20% of 200 is equal to 5% of which of the following?
(A) 4 (B) 50 (C) 80 (D) 200 (E) 800

9. A two-liter bottle of cola costs x cents and a one-liter bottle of cola costs y cents. If two one-liter bottles cost q cents more than one two-liter bottle, which of the following expressions must be true?
(A) $2y + q = x$ (B) $x - q = 2y$ (C) $2y - q = x$ (D) $y - q = x$ (E) $2y - 2x = 2q$

10. Joe the hot dog salesman sold one-half of his supply of hot dogs between 11:15 a.m. and 12:15 p.m. After he sold 18 more between 12:15 p.m. and 1 p.m., he found that he had $\frac{1}{8}$ of the original amount of hot dogs remaining. How many hot dogs did he have at 11:15?
(A) 40 (B) 48 (C) 54 (D) 60 (E) It cannot be determined from the information given.

11. In the given segment, the length of XZ is 48. If XY is represented by 5q and YZ is represented by 7q, what is the length of XY?



(A) 4 (B) 8 (C) 16 (D) 20 (E) 28

12. Joe the hot dog salesman sold 1/2 of his supply of hot dogs between 11:15 a.m. and 12:15 p.m. He sold 1/3 of what he had left between 12:15 p.m. and 1 p.m. If he had 22 hot dogs at 1 p.m., how many hot dogs did he have at 11:15 a.m.?
(A) 4 (B) 66 (C) 88 (D) 99 (E) 132

Answers and an equation that may be used to solve the problem.

1. **C** $8x + 5 = 77$
2. **E** $\frac{1}{3}x = \frac{1}{2}x - 3$
3. **D** $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{6}x + 10 = x$
4. **C** $0.20(x) = 10$
5. **D** $x - \frac{1}{4}x = 48$
6. **C** $M - 6 = W + 6 + 6$
7. **E**
8. **E** $0.2(200) = 0.05x$
9. **C**
10. **B** $x - (1/2)x - 18 = x/8$
11. **D** $5q + 7q = 48$
12. **B** $x - \frac{1}{2}x - \frac{1}{3}(x - \frac{1}{2}x) = 22$

#5 Problems involving percent %

Percent means hundredth.

45% means $\frac{45}{100}$ or 0.45 8% means $\frac{8}{100}$ or 0.08

125% means $\frac{125}{100}$ or 1.25 1000% means $\frac{1000}{100}$ or 10

One way of doing percent problems is to remember the phrase

% of Total = Part

Example 1: What is 30% of 150?

30% of 150 = Part
0.3 (150) = P
45 = P

Example 2: 5 is what percent of 20?

What % of 20 is 5?
P(20) = 5
P = $\frac{5}{20}$ = 0.25 = 25%

Example 3: 56 is 80% of what number?

80% of what number = 56?
0.8 (N) = 56

N = $\frac{56}{0.8}$ = 70

Example 4: After the price of a dog was reduced by 20%, it sold for $40. What was the original price of the dog?

Original price - discount = selling price
Let original price = Q. Q - 0.2(Q) = $40
0.8(Q) = $40

Q = $\frac{\$40}{0.8}$ = $50

Know these fractions and conversions

50% = $\frac{1}{2}$ $33\frac{1}{3}\% = \frac{1}{3}$ $25\% = \frac{1}{4}$ $20\% = \frac{1}{5}$ $60\% = \frac{3}{5}$ $12\frac{1}{2}\%$ or $12.5\% = \frac{1}{8}$

$66\frac{2}{3}\% = \frac{2}{3}$ $75\% = \frac{3}{4}$ $40\% = \frac{2}{5}$ $80\% = \frac{4}{5}$

1. In a certain school, there are 400 boys and 500 girls. If 50% of the boys and 25% of the girls attended a football game, how many students attended the football game?
(A) 300 (B) 325 (C) 400 (D) 500 (E) 675

2. In a certain school, there are 400 boys and 600 girls. If 50% of the boys and 25% of the girls attended a football game, how many students attended the football game?
(A) 35% (B) 37.5% (C) 50% (D) 62.5% (E) 75%

3. A bank offered a yearly interest rate of 6% on money in a savings account. If Joe invested $4,500 at this rate for exactly a year, how much interest did he receive on his investment?
(A) $27 (B) $75 (C) $270 (D) $750 (E) $2700

4. 40% of 50% of 1 is which of the following?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$ (D) $\frac{9}{10}$ (E) $\frac{5}{4}$

5. If a team won 105 games out of a total of 140 played, what percent of the total games played did the team win?
(A) 65% (B) 70% (C) 75% (D) 85% (E) $133\frac{1}{3}\%$

6. If a team won 65 games out of a total of 100 played, what percent of the total games played did the team lose? (There were no ties.)

(A) $33\frac{1}{3}\%$ (B) 35% (C) 65% (D) $66\frac{2}{3}\%$ (E) 75%

7. If a team won 60 games out of a total 90 played, what percent of the total games played did the team lose? (There were no ties.)

(A) 30% (B) $33\frac{1}{3}\%$ (C) 50% (D) 60% (E) $66\frac{2}{3}\%$

8. A man borrowed $2,000 for a year. The cost of the loan was 9% of the amount borrowed. At the end of the year, the man paid back the loan plus the cost of the loan. What was the amount that he paid back?
(A) $2,001.80 (B) $2,018.00 (C) $2,180.00 (D) $3,800.00 (E) $20,000

9. The cost of a dress is $128. Tomorrow it will be on sale at a 10% discount. What will the dress cost when it is on sale?
(A) $12.80 (B) $115.20 (C) $116.20 (D) $118.00 (E) $127.90

10. The sophomore class at a high school has 140 students. Exactly 7 of these students failed one or more courses. What percent of the sophomores failed one or more courses?
(A) 5% (B) 7% (C) 10% (D) 20% (E) 50%

11. If 20 is 5 percent of a number, then what is the number?
(A) 1 (B) 2 (C) 40 (D) 100 (E) 400

12. If twenty people tie for first in a contest and receive equal shares of prize money, what percent of the prize money do twelve of the people together receive?
(A) 12% (B) 20% (C) 24% (D) 40% (E) 60%

13. If 30 pounds of cashew nuts are added to 20 pounds of peanuts, then the peanuts are what percent of the mixture by weight?

(A) 10% (B) 20% (C) $33\frac{1}{3}\%$ (D) 40% (E) $66\frac{2}{3}\%$

14. After a dress was reduced by 25%, its price was \$48. What was the price before the reduction?
(A) \$12 (B) \$60 (C) \$64 (D) \$72 (E) \$192

This is one of the most difficult types of percent problems.
Try to recognize what is happening in the problem.

The price of a dress (D) is reduced by 25% of that price, and the result is \$48.
An equation that represents this is $D - 0.25(D) = 48$.
So $0.75(D) = 48$,

and $D = \frac{48}{0.75} = \mathbf{64}$.

Since the PSAT and SAT are multiple choice tests,
you may also use the answer choices to try to find the correct answer.

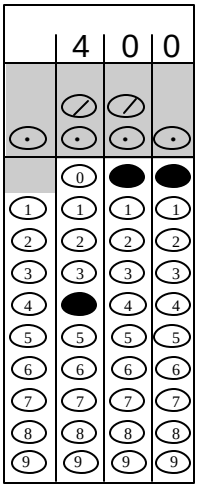
One of the five answers must be correct. Start with choice (E). If \$192 is the original price, find the new price by taking 25% of \$192, which is \$48, and subtracting that amount from \$192. If the result is \$48 (the reduced price given in the problem), then you know that choice (E) is correct. However, when you subtract \$48 from \$192, you get \$144. Therefore, choice (E) is not the correct answer.

Continue this process through the answer choices until you find the one that gives \$48. When choice (D) is tested, the answer is \$54. \$72 - 0.25(\$72) = \$54. When choice (C) is tested, the answer is \$48. \$64 - 0.25(\$64) = \$48. This is the one you are looking for.

Many students will do this problem in the following incorrect manner. Beware of this. They will take 25% of \$48, which is \$12, and add this to \$48. Their answer will be \$60. This is incorrect, but notice that it is choice (B), and it is just waiting to cause you to miss the problem.

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

15. If 25% of x is 100, what is the value of x?



16. Sammy Sosa got 40 hits in April. If his hit production increased by 50% in May, how many hits did Sammy Sosa get in May?

17. Maria made $100 in commissions on Monday. On Tuesday her commissions increased by 50%. On Wednesday her commissions decreased by 50% from **Tuesday's** commissions. How many dollars did Maria make in commissions on Wednesday? (Disregard the $ sign when gridding your answer.)

18. The regular price of a computer is $1,000. It is on sale for 20% off of the regular price. If Mia purchases the computer on sale and pays sales tax of 5% of her purchase price, what is the exact dollar amount that she pays for the computer? (Disregard the $ sign when gridding your answer.)

19. At Lincoln High last year, 0.5% of the senior class were accepted to Ivy League schools. A total of 4 seniors were accepted to these schools. How many students were in the senior class.

Answers: 1. B 2. A 3. C 4. A 5. C 6. B 7. B 8. C 9. B 10. A 11. E 12. E
13. D 14.C 15. 400 16. 60 17. 75 18. 840 19. 800

#6

Inequalities

> <

Examples of inequalities are $x - 3 < 5$ $2x + 4 \geq -8$ $3x < 12 < 4x$

Inequalities are solved just like equations except for one rule.

Whenever you multiply or divide by a negative number, you must reverse the inequality sign.

However, many of the inequality problems do not require the use of algebra. You can get the answer by substituting in values for the variable or variables.

1. If $3x - 4 > 8$, then possible values for x include all except which of the following?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

2. $x > 8$, and $y = 12 - x$, then which of the following must be true?

(A) $y > 4$ (B) $y > 8$ (C) $y > 12$ (D) $y < 4$ (E) $y < 0$

3. For how many integers Q is $1 < 2Q < 5$?

(A) none (B) one (C) two (D) three (E) an infinite number of integers

4. If $4 \leq x \leq 8$ and $5 \leq y \leq 8$, what is the greatest possible value of $x - y$?

(A) 0 (B) 1 (C) 3 (D) 4 (E) 8

5. If $-4 \leq x \leq 7$ and $5 \leq y \leq 7$, what is the least possible value of $x - y$?

(A) 3 (B) -1 (C) -3 (D) -11 (E) -28

6. If $x^2 > y^2$, which of the following is NOT POSSIBLE?
(A) $x > y$
(B) $y > x$
(C) $x > 0$
(D) $y > 0$
(E) $x = y$

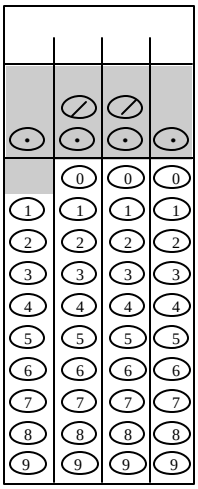
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below. Some problems have many correct answers. If this happens you must enter one possible answer on the grid.

7. If $3 < 2x - 1 < 5$, what is one possible value for x?

8. If $7 < 2x - 1 < 10$, what is one possible value for x?

9. If $3 < 1/x$, and $x \neq 0$, what is one possible value for x?

10. If $x^3 < x$, what is one possible POSITIVE value for x?



11. If $1.2 < r/5 < 1.8$, and r is an integer, what is one possible INTEGER value of r?

Answers: 1. A 2. D 3. C 4. C 5. D 6. E 7. $2 < x < 3$
8. $4 < X < 5.5$ 9. $0 < X < 1/3$ 10. $0 < X < 1$ 11. 7 or 8

#7 **Factors and factoring**

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

You can make some very difficult problems quite easy if you make sure that you learn three common factors.

Notice that there are six expressions in the three equations. They are:

$(x + y)^2$	$(x - y)^2$	$(x + y)(x - y)$
$x^2 + 2xy + y^2$	$x^2 - 2xy + y^2$	$x^2 - y^2$

Whenever you see any of these six in any problem, <u>you should immediately write the expression that it is equal to.</u>

So, if you see $(x + y)^2$, you write $x^2 + 2xy + y^2$.

If you see $x^2 - y^2$, you write $(x + y)(x - y)$.

1. If $x - y = 3$ and $x^2 - y^2 = 3$, what is the value of $x + y$?
(A) 0 (B) 1/2 (C) 1 (D) 1 1/2 (E) 2

Hopefully, after the lecture, you started by writing $x^2 - y^2 = (x + y)(x - y)$.

Answers follow the last problem.

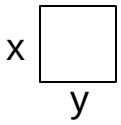
2. $(x - y)^2 + 4xy = ?$

(A) $x^2 - y^2 + 4xy$ (B) $(x - 2y)^2$ (C) $(x - y)^2$ (D) $(x + y)^2$ (E) $(x + 2y)^2$

3. If $x^2 - 9 = r$, $x + 3 = s$, and $rs \neq 0$, then $x - 3 = ?$

(A) $r + s$ (B) $r - s$ (C) $\frac{r + 9}{s}$ (D) rs (E) $\frac{r}{s}$

4. If $x - y = 11$ and $x^2 - y^2 = 165$, then $x + y = ?$
(A) 2 (B) 9 (C) 13 (D) 15 (E) 26

5. For this square 

$(x + y)(x - y) = ?$

(A) 0 (B) $2x$ (C) x^2 (D) $x^2 + 2xy + y^2$ (E) 1

6. If $r = s + 1$ and $s = t^2 - 2t$, what is r in terms of t?

(A) $(t - 1)^2$ (B) $(t + 1)^2$ (C) $t^2 - t$ (D) $t + 1$ (E) $\frac{t^2 - 2t}{t + 1}$

7. $(25)^2 + 2(25)(75) + (75)^2 =$
(A) 6,550 (B) 10,000 (C) 400 (D) 9,900 (E) 3,950

8. If $x + y = 2$ and $x - y = 6$, then what is $x^2 - y^2$?

(A) $\sqrt{12}$ (B) 20 (C) 40 (D) -32 (E) 12

9. If $x + 2y = 6$ and $x - 2y = 8$, then what is $x^2 - 4y^2$?

(A) $4\sqrt{3}$ (B) 45 (C) 48 (D) 14 (E) 292

10. If $x - y = 6$ and $x^2 - y^2 = 12$, then what is $x + y$?

(A) 1/2 (B) 2 (C) 18 (D) 72 (E) $6\sqrt{2}$

11. If $(x + y)^2 = 100$ and $xy = 24$, then what is $x^2 + y^2$?
(A) 100 (B) 52 (C) 76 (D) 10 (E) 576

12. If $(x - y)^2 = 16$ and $xy = 60$, what is $x^2 + y^2$?
(A) 92 (B) 32 (C) 66 (D) 76 (E) 136

13. If $(x - 2)^2 = (x + 2)^2$, then what is the value of x?
(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

Reminder: Learn these expressions and know which pairs are equal.

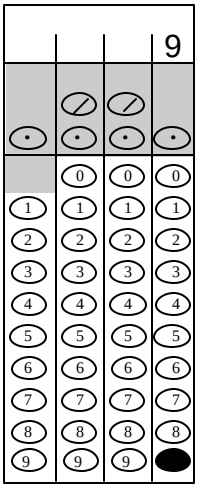
$(x + y)^2$	$(x - y)^2$	$(x + y)(x - y)$
$x^2 + 2xy + y^2$	$x^2 - 2xy + y^2$	$x^2 - y^2$

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

14. If $x - y = 3$ and $x^2 - y^2 = 27$, what is the value of $x + y$?

15. If $x - y = 4$ and $x^2 - y^2 = 24$, what is the value of x?

16. If $x - y = 4$ and and $x + y = 10$, what is the value of $x^2 - y^2$.



17. If $(x + y)^2 = 64$ and $xy = 24$, then what is $x^2 + y^2$?

18. If $(x - y)^2 = 60$ and $xy = 60$, what is $x^2 + y^2$?

Answers: 1. C 2. D 3. E 4. D 5. A 6. A 7. B 8. E 9. C 10. B 11. B 12. E 13. A
14. 9 15. 5 16. 40 17. 16 18. 180

#8

Ratio and proportion

$\frac{A}{B} = \frac{X}{Y}$

A ratio is a comparison of two numbers by division.

Example: If there are 10 boys and 15 girls in a geometry class, the ratio of boys to girls is $\frac{10}{15}$ which is $\frac{2}{3}$. This may also be written 2:3.

The ratio of girls to boys is $\frac{15}{10}$ which is $\frac{3}{2}$. This may be written 3:2.

The ratio of boys to students in the class is $\frac{10}{25}$ which is $\frac{2}{5}$.

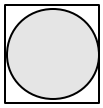
The ratio of girls to students in the class is $\frac{15}{25}$ which is $\frac{3}{5}$.

The single most important thing to do in a problem involving a ratio is to write a fraction using the words of the items in the ratio.

If you are asked for the ratio of boys to girls, you should immediately write: $\frac{\text{Boys}}{\text{Girls}}$

If you are asked for the ratio of cats to dogs, you should immediately write: $\frac{\text{Cats}}{\text{Dogs}}$

If you are asked for the ratio of the



shaded region to the unshaded region, you should immediately write: $\frac{\text{Shaded}}{\text{Unshaded}}$

There is a $1,000,000 fine if you fail to write the ratio with words.

A proportion is an equation involving two ratios.

Example: $\frac{a}{b} = \frac{c}{d}$ (Of course, neither b nor d may be equal to zero.)

You can solve a proportion that has one unknown by cross-multiplying.

If $\frac{4}{12} = \frac{5}{x}$ then $4(x) = 12(5)$

$4x = 60$

$x = 15$

1. If there are exactly 4 red marbles in a jar containing 12 marbles, what is the ratio of red marbles to marbles that are not red?

(A) 1:1 (B) 1:2 (C) 1:3 (D) 1:4 (E) 2:3

2. What is the value of x in the given proportion?

$$\frac{0.8}{0.04} = \frac{0.04}{x}$$

(A) 0.2 (B) 0.02 (C) 0.002 (D) 0.5 (E) 500

3. On a pool table there are fifteen balls, seven that have stripes and eight that are solid in color. If three of the striped balls are removed, what is the ratio of the remaining striped balls to all the balls still on the table?

(A) 7:15 (B) 1:2 (C) 1:3 (D) 4:15 (E) 1:4

4. If $\frac{4}{5} = \frac{x}{4}$, then x = ?

(A) $\frac{5}{16}$ (B) $\frac{5}{4}$ (C) 5 (D) 16 (E) $\frac{16}{5}$

5. A baseball team played twelve games, and there were no ties. All of the following could be the ratio of wins to losses except

(A) 1:1 (B) 1:12 (C) 1:3 (D) 1:2 (E) 1:11

6. The longest hot dog in the world weighs 64 pounds, and it is cut into two pieces. One piece is 90 feet long and weighs 48 pounds. What is the length, in feet, of the whole hot dog?

(A) 30 (B) 60 (C) 90 (D) 120 (E) 270

Explanation: Once you recognize that you do this problem by setting up a ratio, you write

$$\frac{\textbf{length}}{\textbf{weight}} = \frac{\textbf{length}}{\textbf{weight}}$$ and substitute to get

$$\frac{\textbf{90}}{\textbf{48}} = \frac{\textbf{length}}{\textbf{64}}$$ (the 64 is the total weight)

You can solve for the length by cross-multiplying or by just multiplying both sides of the equation by 64.

(90)(64) = (48)(length)

120 = length

7. Two towns are 24 miles apart. On a map, they are 3.6 inches apart. How long, in miles, is a road that measures 0.3 inches on the map?

(A) 0.2 (B) 0.3 (C) 1.0 (D) 1.2 (E) 2

8. If there are 8 boys and 16 girls in a science class, what is the ratio of girls to students in the class?

(A) 1:1 (B) 1:2 (C) 1:3 (D) 2:3 (E) 2:1

9. If the ratio of boys to girls in a class is 2:3 and there are 10 boys in the class, how many girls are in the class?

(A) 10 (B) 15 (C) 20 (D) 23 (E) 25

10. If the ratio of boys to girls in a class is 5:4 and there are 20 boys in the class, how many students are in the class?

(A) 16 (B) 20 (C) 25 (D) 36 (E) 45

To determine the total number of students,
find the number of girls and add that amount to the number of boys.

I hope that you avoided the fine and wrote $\frac{\text{boys}}{\text{girls}} = \frac{5}{4}$

$$\frac{20}{G} = \frac{5}{4}$$

$$(G)(5) = (20)(4)$$
$$5G = 80$$
$$G = 16$$

Since the number of girls, G, in the class is 16, the total number of students in the class is 36.

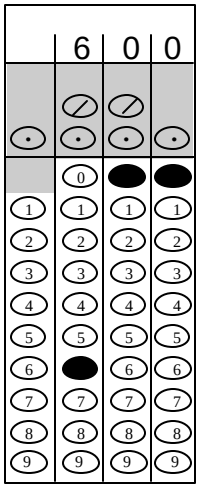
11. If there are 35 students in a mathematics class and 15 of those are boys, what is the ratio of girls to boys in the class?

(A) 3:7 (B) 4:7 (C) 3:4 (D) 4:3 (E) 7:4

Explanation: $\frac{\text{girls}}{\text{boys}} = \frac{20}{15} = \frac{4}{3}$ 4:3

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

12. On a state map, a distance of 80 miles is represented by 1 inch. How many miles are represented by 7.5 inches on the map?



13. Find the value of x if the ratio of 0.25 to x is the same as the ratio of 1.25 to 5.

14. A CD machine produces 240 CDs per hour. At this rate, in how many <u>minutes</u> can the machine produce 48 CDs?

15. If one cup of soup is mixed with two cups of milk to make clam chowder for 4 people, how many cups of soup are needed to make clam chowder for 10 people?

16. It takes Lynn 30 minutes to read a history chapter containing 20 pages. If Meg reads at twice this rate, how many minutes will it take Meg to read a chapter containing 10 pages?

17. The ratio of 9 to 4 is equal to the ratio of 36 to what number?

Answers: 1. B 2. C 3. C 4. E 5. B 6. D 7. E 8. D 9. B 10. D
11. D 12. 600 13. 1 14. 12 15. 2.5 or 5/2 16. 7.5 or 15/2 17. 16

#9 Simplifying expressions

Answers follow the last problem.

1. If $y \neq 0$, $y\left(\frac{1}{y}\right)$ is equal to which of the following?

(A) 0 (B) 1 (C) y (D) $\frac{y+1}{y}$ (E) $\frac{1}{y^2}$

2. If a = 3b, which of the following is equal to 30b - 10a?
(A) 0 (B) b (C) 3b (D) 20b (E) 60b

3. $\frac{1}{2} \div \left(\frac{1}{2} \div \frac{1}{2}\right) = ?$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

4. $\frac{4.8x^9y}{0.3x^3y^3} = ?$

(A) $\frac{0.16x^3}{y^3}$ (B) $\frac{16x^6}{y^3}$ (C) $\frac{16x^6}{y^3}$ (D) $\frac{16x^3}{y^2}$ (E) $\frac{16x^6}{y^2}$

5. If $x \neq 0$, $\frac{(-3x)^3}{-3x^3}$ is equal to which of the following?

(A) -9 (B) -3 (C) 1 (D) 3 (E) 9

6. If $x \neq 0$, $\frac{(-2x)^4}{-2x^4}$ is equal to which of the following?

(A) -8 (B) -4 (C) 1 (D) 4 (E) 8

7. The operation ## is defined by the equation x ## y = x + y - xy.
For example, (-4) ## (2) = -4 + 2 - (-4)(2) = 6.
What is $\left(-\frac{1}{2}\right)$ ## 5 ?

(A) $-\frac{11}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) 7 (E) 8

8. If $(a^x)(a^{10}) = a^{30}$, and $(a^5)^y = a^{30}$, what is the value of x + y?

(A) 9 (B) 15 (C) 20 (D) 26 (E) 45

9. If x is a positive number, which of the following is equal to $\sqrt{36x^2}$?

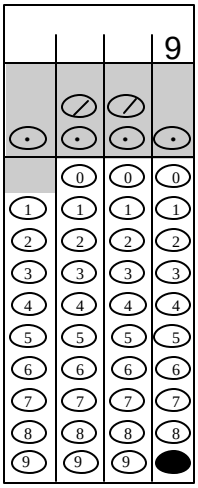
(A) 6x (B) 9x (C) 18x (D) $6\sqrt{x}$ (E) (E) $6x^2$

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

10. If ax + 4 = 9x + 4, what is the value of a?

11. If $(x + 4)^2 = x^2 + bx + c$, what is the value of b + c?

12. If $(x + a)^2 = x^2 + bx + 36$, what is the value of b?



13. My dear Aunt Sally wants to know the value of 24 -12 ÷ 2. What is the correct answer?

14. If $5(0.2x^3 + 2x^2 + 22x + 222) = ax^3 + bx^2 + cx + d$, what is the value of a + b + c + d?

15. What is the value of

$$\frac{\frac{1}{3} + \frac{1}{4}}{\frac{1}{4} + \frac{1}{4}}$$

Answers: 1. B 2. A 3. C 4. E 5. E 6. A 7. D 8. D 9. A 10. 9 11. 24 12. 12 13. 18 14. 1231 15. 7/6 or 1.16 or 1.17

#10

Average

For PSAT and SAT problems that deal with averaging, it is very important to find the total amount of whatever is to be averaged.

1. If a diver receives scores of 9.1, 9.6 and 8.0 in three diving events, what is the average score for these three events?

(A) 8.8 (B) 8.9 (C) 9.0 (D) 9.1 (E) 9.2

Find the total amount of whatever is to be averaged.

In this problem the total is 26.7.
Now divide this number by 3 and get the answer.

2. If the average (arithmetic mean) of -10 and q is -10, what is the value of q?

(A) -20 (B) -10 (C) 0 (D) 10 (E) 20

Find the total amount of whatever is to be averaged.

In this problem this total is -10 + q.
To find the average, divide this quantity (-10 + q) by 2.
Since it is known that the average is -10, set up this equation and solve.

$$\frac{-10 + q}{2} = -10$$

3. The average of three tests was 80. If the average of two of the tests was also 80, what was the score on the third test?

(A) 75 (B) 80 (C) 85 (D) 90
(E) It cannot be determined from the information given.

Find the total amount of whatever is to be averaged.

The total amount of points on the three tests is 240. This amount comes from 3(80)= 240.

The total amount of points on the two tests whose average is 80 is 160. This amount comes from 2(80)= 160.

If two of the tests have a total of 160 points and all three tests have a total of 240 points, the score on the third test is 80, the difference of 240 and 160.

4. The average of three tests was 80. If the average of two of the tests was 75, what was the score on the third test?
(A) 70 (B) 75 (C) 80 (D) 85 (E) 90

5. If 40 is the average of y, y, y, y, 42 and 22, what is the value of y?
(A) 40 (B) 42 (C) 44 (D) 46 (E) 48

Find the total amount of whatever is to be averaged.

In this problem the total is 240. This amount is derived by multiplying the average (40) by the number of items (6).
Two of the items, the 42 and the 22, take up 64 of the 240 total points. This leaves 176 points for the other four items, which are equal to each other. Complete the problem by dividing 176 by 4.

6. There are 30 students in a class that took a test. The average of 10 of the students' grades was 80. The average of the grades of the other 20 students was 89. What was the average grade on the test for all 30 students?
(A) 83 (B) 84 (C) 84.5 (D) 85.5 (E) 86

Find the total amount of whatever is to be averaged.

In this problem the total is 2,580. The 10 students that had an average of 80 would have a total of 800 points. The 20 students that had an average of 89 would have a total of 1,780 points. Thus, the total amount is 2580 points. Divide this by 30 to find the average for all 30 students.

7. If the average of 2q and 3q is 10, what is the value of q?

(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) 8 (E) 10

Find the total amount of whatever is to be averaged.

In this problem the total is 2q + 3q or 5q.

Use the equation $\frac{\text{total amount}}{\text{number of items}}$ = average.

In this problem write $\frac{5q}{2} = 10$ and solve for q.

There is another method to solve this problem that is useful in solving many types of problems. Since the correct answer to the problem must be one of the five choices, q must be one of those answers. Substitute each answer choice for q until the correct one is found.

Answers: 1. B 2. B 3. B 4. E 5. C 6. E 7. C

#11

Problems involving systems of equations

The system of equations questions on the PSAT or SAT deal with two equations that usually have two different variables.

Normally, a system of equations is solved by the substitution method or by the multiplication-addition method.

Substitution Example

$\begin{cases} x = y + 3 \\ y + 2x = 3 \end{cases}$

Substitute in $y + 2x = 3$
and get $y + 2(y+3) = 3$
simplify $y + 2y + 6 = 3$
$3y + 6 = 3$
subtract 6 from both sides $\underline{- 6 = -6}$
$3y = -3$
and $y = -1$

Now, to find x, substitute -1 for y in either original equation.

$x = -1 + 3$
$x = 2$
So the answer is (2, -1) .

Multiplication-Addition Example

$\begin{cases} y + 2x = 6 \\ 2y - x = 2 \end{cases}$

Multiply the bottom equation by 2. This will create a situation in which there is a 2x in the top equation and a -2x in the bottom equation.

$\begin{cases} y + 2x = 6 \\ 4y - 2x = 4 \end{cases}$

When these two equations are added together the result is
$5y = 10$
$y = 2$
If y = 2, then x = 2, and the answer is **(2,2)**

On the PSAT or SAT many times it is not necessary solve the whole system.

Example: If A + B - C = 8
and A + B + C = 7,
what is the value of 2A + 2B?

I hope you can see that if you just add the two equations together,
the result is 2A + 2B = 15.

1. If a - b = 3 and a + b = 5, what is the value of 4a?
(A) 4 (B) 8 (C) 12 (D) 16 (E) 32

2. If 3x - 4y = 4 and -2x + 5y = 2, what is the value of x + y?
(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

3. If 3x - 4y = -2 and -2x + 5y = 5, what is the value of 5(x +y)?
(A) 3 (B) 9 (C) 15 (D) 21 (E) 35

4. If 4x + 3y = 10 and 3x + 2y = 7, what is the value of 4(x +y)?
(A) 3 (B) 6 (C) 9 (D) 12 (E) 68
Hint: subtract the second equation from the first one, and you will find x + y.

5. If $x + y = 6$ and $x + 2y = 8$, what is the value of $y - x$?
(A) -6 (B) -2 (C) 0 (D) 2 (E) 22

6. If the sum of two numbers is 9 and the product of the same two numbers is 20, what is the smallest of the two numbers?
(A) -5 (B) -4 (C) 0 (D) 4 (E) 5

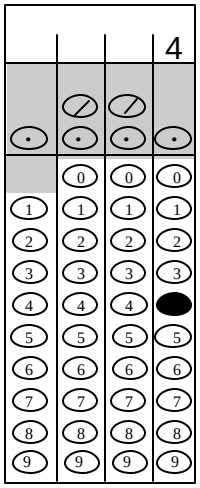
7. If $3x = 7$ and $5y - 3x = 13$, what is the value of y?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 20

8. If $x = 3 + 5 + 7 + 9 + y$ and $y = \frac{1}{4}x$, what is the value of y?

(A) 4 (B) 6 (C) 8 (D) 12 (E) 32

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

9. If $a + b + c = 5$ and $a + b - c = 3$. what is the value of $a + b$?



10. If $2x + 5y = 7$ and $x + 3y = 1$. what is the value of $x + 2y$?

11. From the following equations, determine the value of $a + b$. (If you want a hint, see the small upside down print below.)

$a + 2b + c + d = 14$
$2a - b - c + d = 1$
$-a + b - c - d = -2$
$-a + b + c - d = 2$

add the four equations together

Answers: 1. D 2. C 3. C 4. D 5. B 6. D 7. C 8. C
9. 4 10. 6 11. 7.5 or 15/2

#12

Slope

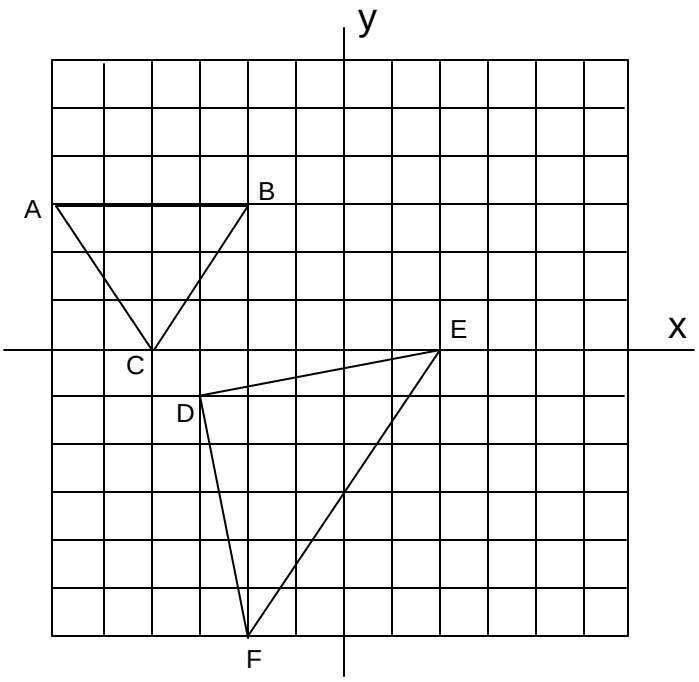
$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Expect at least one slope question on any PSAT or SAT. Remember the following:

1) parallel lines have the same slope
2) perpendicular lines have slopes that are negative reciprocals

1. In the given figure, which two triangle sides have the same slope?

(A) AB and DE
(B) BC and EF
(C) AC and DF
(D) CB and DE
(E) AC and BC



2. In the figure above, point M (not shown) is found by starting at E and moving 2 units up and then moving 4 units to the right. What is the slope of segment EM?

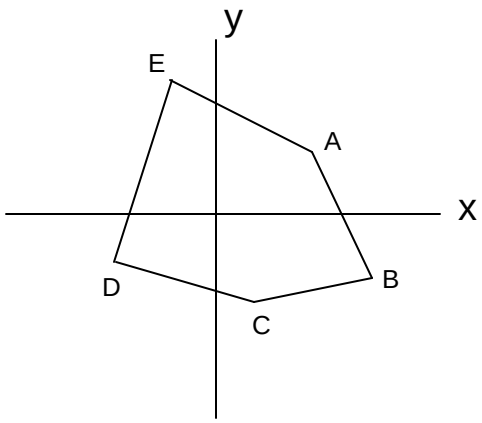
(A) 1/4 (B) 1/2 (C) 1 (D) 2 (E) 4

3. What is the slope of a line that passes through the origin (0, 0) and the point (1, -2)?

(A) -2 (B) -1/2 (C) -1 (D) 1/2 (E) 2

4. Pentagon ABCDE is placed on the coordinate plane. Which side of the pentagon is the segment with the greatest slope?

(A) AB (B) BC (C) CD (D) DE (E) AE

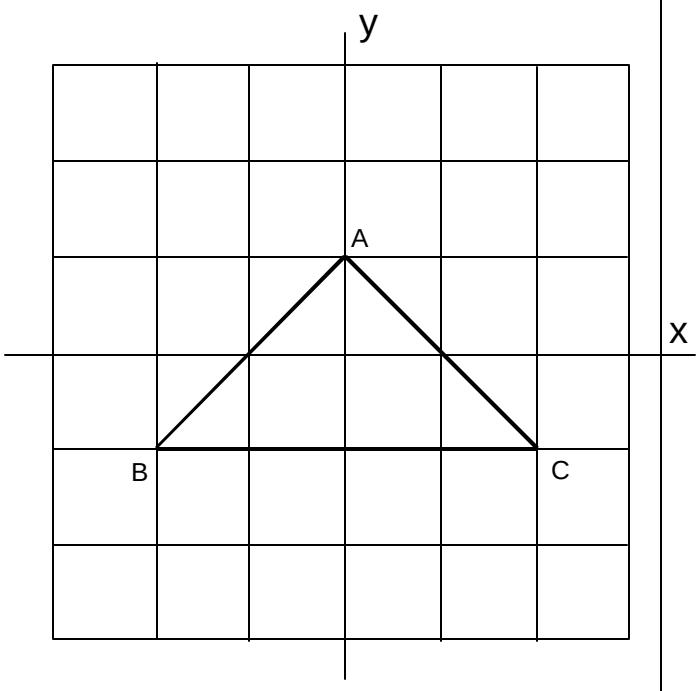


5. Three lines are on a coordinate plane. Line *l* has a slope of 0 and passes through (2, 5). Line *q* has a slope of 0 and passes through (3, 6). Line *r* has a slope of 1 and passes through (4, 4). How many points of intersection are there for these lines.

(A) 0 (B) 1 (C) 2 (D) 3 (E) More than 3

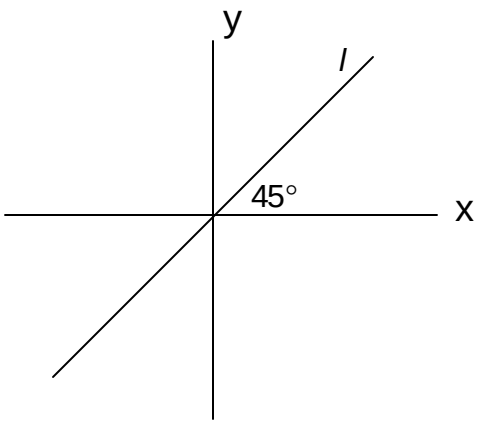
6. What is the sum of the slopes of the three sides of triangle ABC

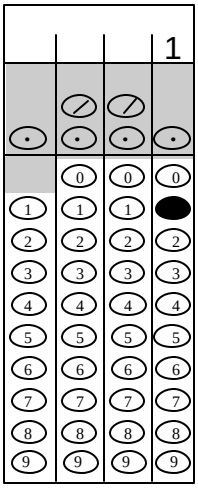
(A) 0
(B) 1
(C) 2
(D) 4
(E) 8



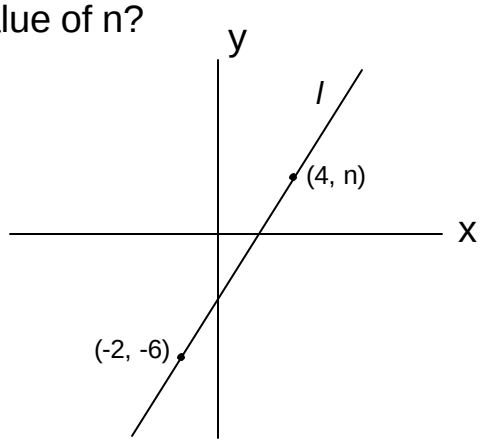
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

7. In the given figure, what is the slope of line *l*?





8. In the given figure, the slope of line *l* is 3/2. What is the value of n?



9. Line *l* passes through the origin (0, 0) and BETWEEN points (1, 1) and (1, 2). What is one possible value for the slope of line *l* ?

10. Line *q* has a slope of 0 and passes through points (1, 4) and (3, k). What is the value of k?

Answers: 1. B 2. B 3. A 4. D 5. C 6. A 7. 1 8. 3 9. $1 < m < 2$ 10. 4